Dynamic investment strategies and rebalancing gains

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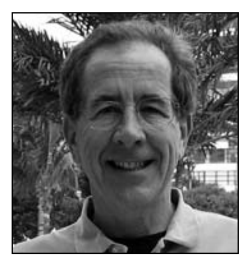
Let's assume that we live in a stylised world where every investment vehicle has zero expected return and 30% annual volatility. Furthermore, suppose the nature of the environment is truly random, so even the best astrologist cannot foretell the future. In such a hostile world, investors do not have many positive choices. However, if certain conditions are met, we can construct an investment strategy that will provide a 4% expected return with a reasonable level of risk.

The key concept to such a strategy is to adopt a multi-period framework for portfolio construction. In contrast, when the traditional (buy-and-hold) strategy is employed, the portfolio weights are set at the beginning and no rebalancing occurs until the end of the investment horizon. The multi-period models allow us to rebalance the portfolio components during the period. Among many others, we illustrate a simple, yet efficient approach to rebalancing – the fixed mix rebalancing rule. We first formalise this rule so that the primary insight can be easily revealed. Suppose there are n tradable assets and asset i follows geometric Brownian motion with expected log return r_i and volatility σ_i. Also suppose the correlation between asset i and asset j is ρ_{ij}. Given the weight on asset i w_i, it can be shown that the log-return of the portfolio is normally distributed. That is,

$r_{Pfo} \sim N\left(\mu_{pfo}, \sigma^2_{pfo}\right)$, where

$$\mu_{pfo} = \sum_{i=1}^{n} w_i r_i + \frac{1}{2}\sum_{i=1}^{n} w_i \sigma_i^2 - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \rho_{ij} \sigma_i \sigma_j \qquad (1)$$

$$\sigma^2_{pfo} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \rho_{ij} \sigma_i \sigma_j ,$$



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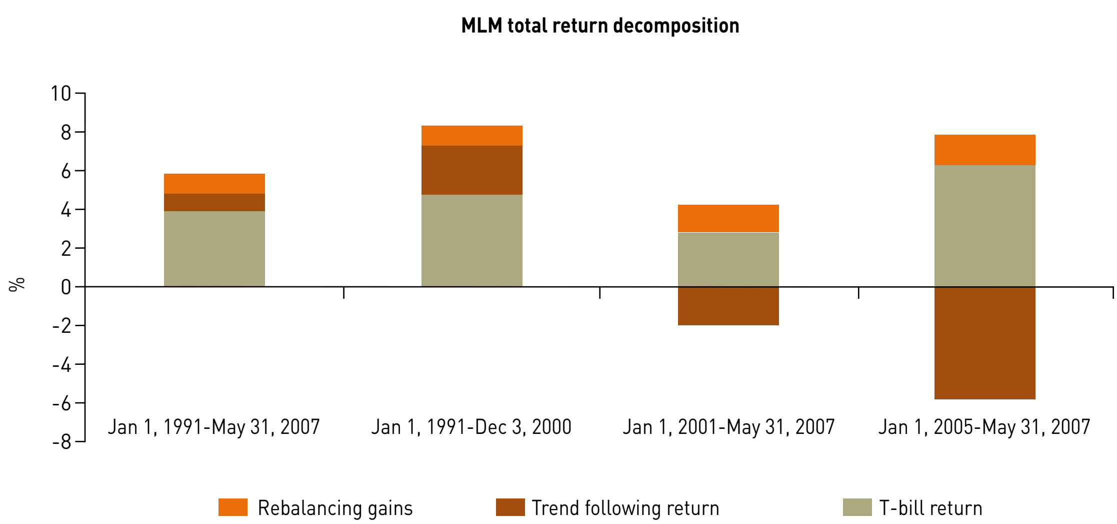
given the fixed-mix rebalancing rule. Here, the fixed mix rule requires that the portfolio is rebalanced continuously so that the weight on asset *i* is fixed to w_i at all time junctures. For instance, as the price of an asset goes up, its weight should decrease accordingly. Then, the investor continuously buys/sells assets so that the weights are equal to the initial settings (see [1-3] for the complete discussion).

In (1), the first term in μ_{pfo} $(\Sigma_i w_i r_i)$ represents the buy-and-hold return, whereas the second and third terms $(\Sigma_i w_i \sigma_i^2/2 - \Sigma_i \Sigma_j w_i w_j \rho_{ij} \sigma_i \sigma_j/2)$, depict the extra return due to applying the fixed-mix rule, called 'rebalancing gains' or 'volatility pumping'. The latter part is positive, unless all assets are perfectly correlated (ρ_{ij}=1). In fact, the expected return of the portfolio becomes larger 1) as correlations (ρ_{ij}) decrease; and 2) as volatilities (σ_i) increase. Therefore, an investment vehicle with a low return and a high volatility can play a significant role. As long as the vehicle has low correlations with other assets, it can 'pump up' the portfolio's expected return without worsening volatility. In this context, while it might be valid for an overall portfolio, the Sharpe ratio may not be an appropriate performance measure for a single investment vehicle. In other words, a volatile asset should not be penalised if its return is positive and it is not highly correlated with other assets, since it can serve as a medium for the rebalancing gains as well as a novel source of diversification.

Next, we apply the fixed-mix rule to our stylised world. For simplicity, let's assume that there are 10 independent assets and they are equally weighted in our portfolio. Then, the portfolio return is normally distributed with a 4.275% expected return and a 9.5% volatility. What a pleasant result in such a difficult environment!

There are a number of applications of rebalancing gains in practical settings. A significant example involves the Mount Lucas Management (MLM) Index [4]. It is an equally weighted, monthly rebalanced investment in 25 futures contracts in commodity, fixed income, and currency markets. Briefly, the monthly positions (long or short) are determined by trend following strategies. The total return of the MLM Index can be decomposed into three parts.

Decomposition of MLM index returns for different time periods **Exhibit 1**



Source:

The first one is the T-bill return gained from the capital allocated for margin requirements. The second component is generated by trend following the futures prices. The third component, rebalancing gains, is earned when all markets are invested with equal weights at the beginning of each month. If trend following strategy had been applied to all the markets without reweighing at each month, then there would be no rebalancing gains. Exhibit 1 shows how those three components affected the total return of the MLM Index for selected time periods. Trend following has underperformed for the last several years. Still, rebalancing provided positive returns over the recent period. This shows how periodic reallocation of capital among the markets boosts the performance of a long term investment strategy with the contribution of rebalancing gains.

Several potential obstacles related to the fixed-mix approach should be addressed. First, it is getting harder to find independent or low correlated assets. For instance, oil and corn, which were once thought to be relatively independent, are now highly correlated, because ethanol is manufactured from corn. Also, due to globalisation, the correlation of assets across countries is becoming higher.

Second, even if we are successful in finding a set of independent assets with positive returns and high volatilities, independence is likely to disappear under extreme conditions; there is considerable evidence that stock correlations dramatically increase when the market crashes. Furthermore, it is well-known that stock returns and volatilities are negatively correlated.

Third, since the fixed-mix model requires portfolio rebalancing, one must consider transaction costs, such as capital gain taxes. Such costs not only deteriorate the investment performance but also make it harder to implement the model. See Mulvey and Simsek and other references [5-9] for concepts related to optimising a multi-period portfolio with transaction costs.

In summary, a multi-period portfolio model provides significant advantages over the traditional single-period Markowitz model. For example, many real-world temporal issues can be addressed, such as transaction costs, time-based goals and liabilities, and savings/contribution decisions. Importantly, active rebalancing strategies, such as the fixed-mix rule, can outperform the buy-and-hold approach under selected conditions. Many other dynamic strategies are possible; they can be evaluated via a multi-period framework. Future research should be aimed at developing improved (robust/efficient) rules for rebalancing an investor's portfolio within a multi-period model.

References:

1. Luenberger D. Investment Science; Oxford University Press: New York, 1997.
2. Fernholz R. Stochastic Portfolio Theory; Springer-Verlag: New York, 2002.
3. Mulvey J.M., Ural C., Zhang Z. (2007) Improving performance for long-term investors: wide diversification, leverage, and overlay strategies. *Quantitative Finance*. 7.2:1-13.
4. Mulvey J.M., Kaul S.S.N. and Simsek KD. (2004) Evaluating a trend-following commodity index for multi-period asset allocation. *Journal of Alternative Investments*. summer:54-69.
5. Mulvey, J.M. and Simsek, K.D. Rebalancing strategies for long-term investors. In Kontoghiorghes, E.J., Rustem, B. and Siokos, S. ed., Computational Methods in Decision-Making, Economics and Finance: Optimization Models. Kluwer, 2002, 15-33.
6. Davis, M.H.A. and Norman, A.R. (1990). Portfolio selection with transaction costs. Mathematics of Operations Research. 15:676-713.
7. Dumas, B. and Luciano, E. (1991). An exact solution to a dynamic portfolio choice problem under transaction costs. *Journal of Finance*. 46:577-595.
8. Fernholz, R. and Shay, B. (1982). Stochastic portfolio theory and stock market equilibrium. *Journal of Finance*. 37:615-624.
9. Shreve, S.E. and Soner, H.M. (1991). Optimal investment and consumption with two bonds and transaction costs. *Mathematical Finance*. 1:53-84.

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